General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MFP2

Unit Further Pure 2

Thursday 6 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 (a) Sketch on an Argand diagram the locus of points satisfying the equation

$$|z - 6\mathrm{i}| = 3$$ *(3 marks)*

(b) It is given that z satisfies the equation $|z - 6\mathrm{i}| = 3$.

(i) Write down the greatest possible value of $|z|$. *(1 mark)*

(ii) Find the greatest possible value of $\arg z$, giving your answer in the form $p\pi$, where $-1 < p \leqslant 1$. *(3 marks)*

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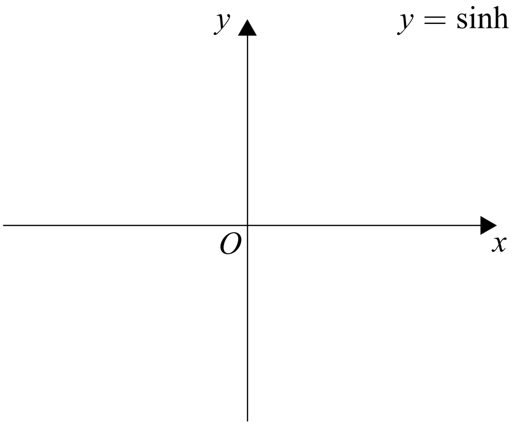
2 (a) (i) Sketch on the axes below the graphs of $y = \sinh x$ and $y = \cosh x$. *(3 marks)*

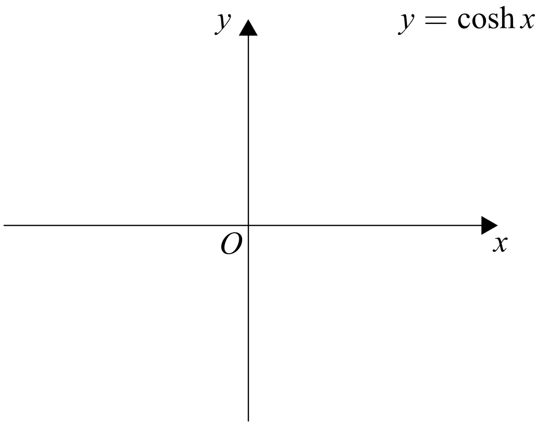
(ii) Use your graphs to explain why the equation

$$(k + \sinh x)\cosh x = 0$$

where k is a constant, has exactly one solution. *(1 mark)*

(b) A curve C has equation $y = 6\sinh x + \cosh^2 x$. Show that C has only one stationary point and show that its y-coordinate is an integer. *(5 marks)*





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3 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2\,, \qquad u_{n+1} = \frac{5u_n - 3}{3u_n - 1}$$

Prove by induction that, for all integers $n \geqslant 1$,

$$u_n = \frac{3n + 1}{3n - 1}$$ *(6 marks)*

4 (a) Given that $f(r) = r^2(2r^2 - 1)$, show that

$$f(r) - f(r-1) = (2r-1)^3$$ *(3 marks)*

(b) Use the method of differences to show that

$$\sum_{r=n+1}^{2n} (2r-1)^3 = 3n^2(10n^2 - 1)$$ *(4 marks)*

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5 The cubic equation

$$z^3 + pz^2 + qz + 37 - 36\mathrm{i} = 0$$

where p and q are constants, has three complex roots, α, β and γ.

It is given that $\beta = -2 + 3\mathrm{i}$ and $\gamma = 1 + 2\mathrm{i}$.

(a) (i) Write down the value of $\alpha\beta\gamma$. *(1 mark)*

(ii) Hence show that $(8 + \mathrm{i})\alpha = 37 - 36\mathrm{i}$. *(2 marks)*

(iii) Hence find α, giving your answer in the form $m + n\mathrm{i}$, where m and n are integers. *(3 marks)*

(b) Find the value of p. *(1 mark)*

(c) Find the value of the complex number q. *(2 marks)*

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6 (a) Show that $\dfrac{1}{5\cosh x - 3\sinh x} = \dfrac{\mathrm{e}^x}{m + \mathrm{e}^{2x}}$, where m is an integer. *(3 marks)*

(b) Use the substitution $u = \mathrm{e}^x$ to show that

$$\int_0^{\ln 2} \frac{1}{5\cosh x - 3\sinh x}\,\mathrm{d}x = \frac{\pi}{8} - \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$$ *(5 marks)*

Turn over ▶

7 (a) (i) Show that

$$\frac{\mathrm{d}}{\mathrm{d}u}\left(2u\sqrt{1+4u^2}+\sinh^{-1}2u\right)=k\sqrt{1+4u^2}$$

where k is an integer. *(4 marks)*

(ii) Hence show that

$$\int_0^1 \sqrt{1+4u^2}\,\mathrm{d}u = p\sqrt{5}+q\sinh^{-1}2$$

where p and q are rational numbers. *(2 marks)*

(b) The arc of the curve with equation $y=\frac{1}{2}\cos 4x$ between the points where $x=0$ and $x=\frac{\pi}{8}$ is rotated through 2π radians about the x-axis.

(i) Show that the area S of the curved surface formed is given by

$$S=\pi\int_0^{\frac{\pi}{8}}\cos 4x\sqrt{1+4\sin^2 4x}\,\mathrm{d}x$$ *(2 marks)*

(ii) Use the substitution $u=\sin 4x$ to find the exact value of S. *(4 marks)*

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8 (a) (i) Use de Moivre's theorem to show that

$$\cos 4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$$

and find a similar expression for $\sin 4\theta$. *(5 marks)*

(ii) Deduce that

$$\tan 4\theta=\frac{4\tan\theta-4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$$ *(3 marks)*

(b) Explain why $t=\tan\frac{\pi}{16}$ is a root of the equation

$$t^4+4t^3-6t^2-4t+1=0$$

and write down the three other roots in trigonometric form. *(4 marks)*

(c) Hence show that

$$\tan^2\frac{\pi}{16}+\tan^2\frac{3\pi}{16}+\tan^2\frac{5\pi}{16}+\tan^2\frac{7\pi}{16}=28$$ *(5 marks)*